Section 6.3
Antidifferentiation by Parts

Product Rule for Differentiation

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Product Rule in Integral Form

$$\int\left(u\frac{dv}{dx}\right)dx = \int\left(\frac{d}{dx}uv\right)dx - \int\left(v\frac{du}{dx}\right)dx$$

Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du$$

Example

Evaluate $\int x \cos x \, dx =$

$$\int x\, e^{x}\, dx =$$

$$\int x^2 \ln x \, dx =$$

$$\int \ln x \, dx =$$

$$\int x^2 e^x \, dx =$$

The Order

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Evaluate

$$\int e^x \cos x \, dx$$

$$\int 2x\, e^{3x}\, dx$$

$$\int x^2 \cos 3x \, dx$$

$$\int e^{2x} \sin x \, dx =$$

$$\int x \csc^2 x \, dx$$

$u = x \qquad dv = \csc^2 x \, dx$

$du = dx \qquad v = -\cot x$

$$= -x\cot x - \int -\cot x \, dx$$

$$= -x\cot x + \int \cot x \, dx$$

$$= -x\cot x + \int \frac{\cos x}{\sin x} \, dx$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$$\int e^{3x} \cos x \, dx =$$

$$\int e^x \cos 2x \, dx =$$

Tabular Integration

- Shortcut for Integration by Parts

Only if u is a polynomial

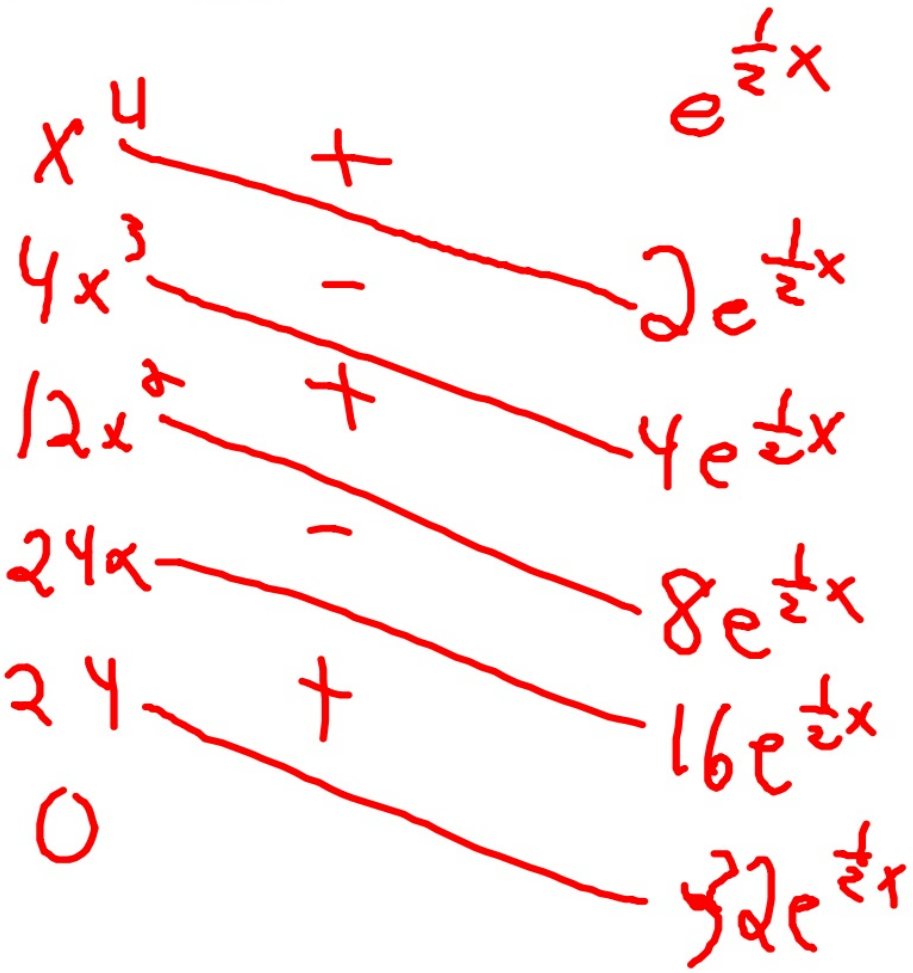
Evaluate $\int x^2 e^x \, dx$

With $f(x) = x^2$ and $g(x) = e^x$, we list:

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	$(+)$ →	e^x
$2x$	$(-)$ →	e^x
2	$(+)$ →	e^x
0	→	e^x

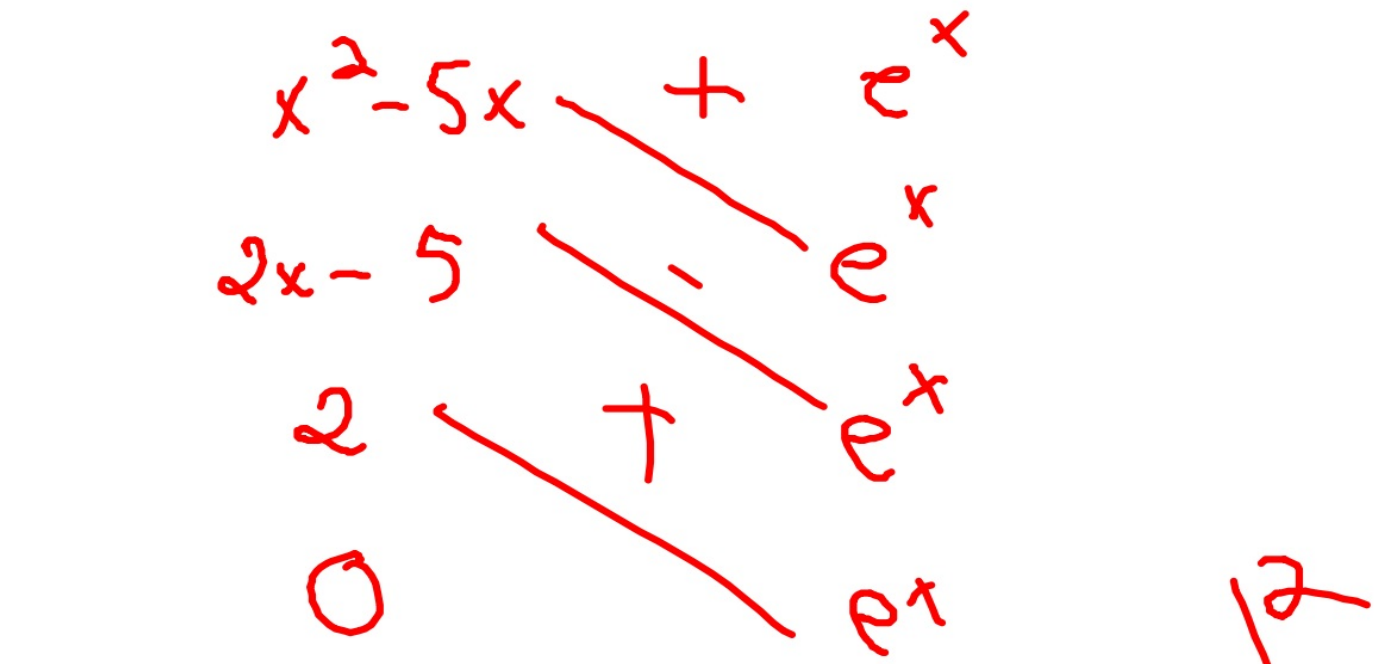
$$\int x^4 e^{3x}\, dx =$$

$$\int x^4 e^{(1/2)x}\,dx$$



$$\int x^3 \sin x \, dx$$

$$\int_0^2 (x^2 - 5x)\, e^x \, dx =$$



$$e^x(x^2 - 5x) - e^x(2x - 5) + e^x(2)\Big|_0^2$$

$$\int \frac{x^2 - 9}{x} \, dx$$

$$\int \frac{x}{x^2 - 9}\, dx$$

Find

$$\int \sin^{-1}x \, dx =$$

$u = \sin^{-1}x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad v = x$

$= x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$y = 1 - x^2$

$\frac{dy}{dx} = -2x \qquad \frac{dy}{-2} = x\,dx$

$dy = -2x\,dx$